OCaml$_{light}$ in Ott

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OCaml$_{light}$ (ESOP '08) is a formal semantics for a substantial subset of the Objective Caml core language, suitable for writing and verifying real programs.

OCaml$_{light}$ key points

- Written in Ott
- Faithful to Objective Caml (very nearly)
- Type soundness proof mechanized in HOL (Coq and Isabelle/HOL definitions generated too)
- Operational semantics validated on test programs
- Small-step operational semantics (131 rules)
- Type system (179 rules, below)

- definitions:
  - variant data types (e.g., `type t = I of int | C of char`),
  - record types (e.g., `type t = {f : int; g : bool}`),
  - parametric type constructors (e.g., `type 'a t = C of 'a`),
  - type abbreviations (e.g., `type 'a t = 'a * int`),
  - mutually recursive combinations of the above (excepting abbreviations),
  - exceptions, and values;
- expressions for type annotations, sequencing, and primitive values (functions, lists, tuples, and records);
- `with` (record update), `if`, `while`, `for`, `assert`, `try`, and `raise` expressions;
- let-based polymorphism with an SML-style value restriction;
- mutually-recursive function definitions via `let rec`;
- pattern matching, with nested patterns, `as` patterns, and "or" (`|`)patterns;
- mutable references with `ref`, `!`, and `:=`;
- polymorphic equality (the Objective Caml = operator);
- 31-bit word semantics for `int`s (using an existing HOL library); and
- IEEE-754 semantics for `float`s (using an existing HOL library).

The OCaml$_{light}$ Operational Semantics (131 rules)

$\boxed{\vdash \mathit{expr}\ \mathbf{matches}\ \mathit{pattern}}$ **Pattern matching**

$$\frac{}{\vdash v\ \mathbf{matches}\ x}\quad\text{JM\_matchP\_var}$$

$$\frac{}{\vdash v\ \mathbf{matches}\ \_}\quad\text{JM\_matchP\_any}$$

$$\frac{}{\vdash \mathit{constant}\ \mathbf{matches}\ \mathit{constant}}\quad\text{JM\_matchP\_constant}$$

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat}}{\vdash v\ \mathbf{matches}\ \mathit{pat}\ \mathbf{as}\ x}\quad\text{JM\_matchP\_alias}$$

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat}}{\vdash v\ \mathbf{matches}\ (\mathit{pat} : t)}\quad\text{JM\_matchP\_typed}$$

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat}_1}{\vdash v\ \mathbf{matches}\ \mathit{pat}_1 \mid \mathit{pat}_2}\quad\text{JM\_matchP\_or\_left}$$

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat}_2}{\vdash v\ \mathbf{matches}\ \mathit{pat}_1 \mid \mathit{pat}_2}\quad\text{JM\_matchP\_or\_right}$$

$$\frac{\vdash v_1\ \mathbf{matches}\ \mathit{pat}_1 \quad \ldots \quad \vdash v_n\ \mathbf{matches}\ \mathit{pat}_n}{\vdash \mathit{constr}\,(v_1, \ldots, v_n)\ \mathbf{matches}\ \mathit{constr}\,(\mathit{pat}_1, \ldots, \mathit{pat}_n)}\quad\text{JM\_matchP\_construct}$$

$$\frac{}{\vdash \mathit{constr}\,(v_1, \ldots, v_n)\ \mathbf{matches}\ \mathit{constr}\ \_}\quad\text{JM\_matchP\_construct\_any}$$

$$\frac{\vdash v_1\ \mathbf{matches}\ \mathit{pat}_1 \quad \ldots. \quad \vdash v_n\ \mathbf{matches}\ \mathit{pat}_n}{\vdash (v_1, \ldots., v_n)\ \mathbf{matches}\ (\mathit{pat}_1, \ldots., \mathit{pat}_n)}\quad\text{JM\_matchP\_tuple}$$

$$\frac{\begin{array}{l}\mathit{field\_name}'_1 = v'_1 \ldots \mathit{field\_name}'_n = v'_n\, \mathit{fn}_1 = v''_1 \,..\, \mathit{fn}_l = v''_l\ \mathbf{PERMUTES}\ \mathit{field\_name}_1 = v_1 \ldots \mathit{field\_name}_m = v_m \\ \vdash v'_1\ \mathbf{matches}\ \mathit{pat}_1 \quad \ldots \quad \vdash v'_n\ \mathbf{matches}\ \mathit{pat}_n \\ \mathit{field\_name}_1 \ldots \mathit{field\_name}_m\ \mathbf{distinct}\end{array}}{\vdash \{\mathit{field\_name}_1 = v_1 ; \ldots ; \mathit{field\_name}_m = v_m\}\ \mathbf{matches}\ \{\mathit{field\_name}'_1 = \mathit{pat}_1 ; \ldots ; \mathit{field\_name}'_n = \mathit{pat}_n\}}\quad\text{JM\_matchP\_record}$$

$$\frac{\begin{array}{l}\vdash v_1\ \mathbf{matches}\ \mathit{pat}_1 \\ \vdash v_2\ \mathbf{matches}\ \mathit{pat}_2\end{array}}{\vdash v_1 :: v_2\ \mathbf{matches}\ \mathit{pat}_1 :: \mathit{pat}_2}\quad\text{JM\_matchP\_cons}$$

$\boxed{\vdash \mathit{expr}\ \mathbf{matches}\ \mathit{pattern} \ \rhd\ \{\!\{\mathit{substs\_x}\}\!\}}$ **Pattern matching with substitution creation**

$$\frac{}{\vdash v\ \mathbf{matches}\ x \ \rhd\ \{\!\{x \leftarrow v\}\!\}}\quad\text{JM\_match\_var}$$

$$\frac{}{\vdash v\ \mathbf{matches}\ \_ \ \rhd\ \{\!\{\ \}\!\}}\quad\text{JM\_match\_any}$$

$$\frac{}{\vdash \mathit{constant}\ \mathbf{matches}\ \mathit{constant} \ \rhd\ \{\!\{\ \}\!\}}\quad\text{JM\_match\_constant}$$

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat} \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_n \leftarrow v_n\}\!\}}{\vdash v\ \mathbf{matches}\ \mathit{pat}\ \mathbf{as}\ x \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_n \leftarrow v_n, x \leftarrow v\}\!\}}\quad\text{JM\_match\_alias}$$

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat} \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_n \leftarrow v_n\}\!\}}{\vdash v\ \mathbf{matches}\ (\mathit{pat} : t) \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_n \leftarrow v_n\}\!\}}\quad\text{JM\_match\_typed}$$

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat}_1 \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_n \leftarrow v_n\}\!\}}{\vdash v\ \mathbf{matches}\ \mathit{pat}_1 \mid \mathit{pat}_2 \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_n \leftarrow v_n\}\!\}}\quad\text{JM\_match\_or\_left}$$

$$\frac{\begin{array}{l}\neg(v\ \mathbf{matches}\ \mathit{pat}_1) \\ \vdash v\ \mathbf{matches}\ \mathit{pat}_2 \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_n \leftarrow v_n\}\!\}\end{array}}{\vdash v\ \mathbf{matches}\ \mathit{pat}_1 \mid \mathit{pat}_2 \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_n \leftarrow v_n\}\!\}}\quad\text{JM\_match\_or\_right}$$

$$\frac{\vdash v_1\ \mathbf{matches}\ \mathit{pat}_1 \ \rhd\ \{\!\{\mathit{substs\_x}_1\}\!\} \quad \ldots \quad \vdash v_n\ \mathbf{matches}\ \mathit{pat}_n \ \rhd\ \{\!\{\mathit{substs\_x}_n\}\!\}}{\vdash \mathit{constr}\,(v_1, \ldots, v_n)\ \mathbf{matches}\ \mathit{constr}\,(\mathit{pat}_1, \ldots, \mathit{pat}_n) \ \rhd\ \{\!\{\mathit{substs\_x}_1 @ \ldots @ \mathit{substs\_x}_n\}\!\}}\quad\text{JM\_match\_construct}$$

$$\frac{}{\vdash \mathit{constr}\,(v_1, \ldots, v_n)\ \mathbf{matches}\ \mathit{constr}\ \_ \ \rhd\ \{\!\{\ \}\!\}}\quad\text{JM\_match\_construct\_any}$$

$$\frac{\vdash v_1\ \mathbf{matches}\ \mathit{pat}_1 \ \rhd\ \{\!\{\mathit{substs\_x}_1\}\!\} \quad \ldots. \quad \vdash v_n\ \mathbf{matches}\ \mathit{pat}_n \ \rhd\ \{\!\{\mathit{substs\_x}_n\}\!\}}{\vdash (v_1, \ldots., v_n)\ \mathbf{matches}\ (\mathit{pat}_1, \ldots., \mathit{pat}_n) \ \rhd\ \{\!\{\mathit{substs\_x}_1 @ \ldots. @ \mathit{substs\_x}_n\}\!\}}\quad\text{JM\_match\_tuple}$$

$$\frac{\begin{array}{l}\mathit{field\_name}'_1 = v'_1 \ldots \mathit{field\_name}'_n = v'_n\, \mathit{fn}_1 = v''_1 \,..\, \mathit{fn}_l = v''_l\ \mathbf{PERMUTES}\ \mathit{field\_name}_1 = v_1 \ldots \mathit{field\_name}_m = v_m \\ \vdash v'_1\ \mathbf{matches}\ \mathit{pat}_1 \ \rhd\ \{\!\{\mathit{substs\_x}_1\}\!\} \quad \ldots \quad \vdash v'_n\ \mathbf{matches}\ \mathit{pat}_n \ \rhd\ \{\!\{\mathit{substs\_x}_n\}\!\} \\ \mathit{field\_name}_1 \ldots \mathit{field\_name}_m\ \mathbf{distinct}\end{array}}{\vdash \{\mathit{field\_name}_1 = v_1 ; \ldots ; \mathit{field\_name}_m = v_m\}\ \mathbf{matches}\ \{\mathit{field\_name}'_1 = \mathit{pat}_1 ; \ldots ; \mathit{field\_name}'_n = \mathit{pat}_n\} \ \rhd\ \{\!\{\mathit{substs\_x}_1 @ \ldots @ \mathit{substs\_x}_n\}\!\}}\quad\text{JM\_match\_record}$$

$$\frac{\begin{array}{l}\vdash v_1\ \mathbf{matches}\ \mathit{pat}_1 \ \rhd\ \{\!\{\mathit{substs\_x}_1\}\!\} \\ \vdash v_2\ \mathbf{matches}\ \mathit{pat}_2 \ \rhd\ \{\!\{\mathit{substs\_x}_2\}\!\}\end{array}}{\vdash v_1 :: v_2\ \mathbf{matches}\ \mathit{pat}_1 :: \mathit{pat}_2 \ \rhd\ \{\!\{\mathit{substs\_x}_1 @ \mathit{substs\_x}_2\}\!\}}\quad\text{JM\_match\_cons}$$

$\boxed{\mathbf{recfun}\,(\mathit{letrec\_bindings}, \mathit{pattern\_matching}) \ \rhd\ \mathit{expr}}$ **Recursive function helper**

$$\frac{\mathit{letrec\_bindings} = (x_1 = \mathbf{function}\ \mathit{pattern\_matching}_1\ \mathbf{and} \ldots \mathbf{and}\ x_n = \mathbf{function}\ \mathit{pattern\_matching}_n)}{\mathbf{recfun}\,(\mathit{letrec\_bindings}, \mathit{pattern\_matching}) \ \rhd\ \{\!\{x_1 \leftarrow \mathbf{let\ rec}\ \mathit{letrec\_bindings}\ \mathbf{in}\ x_1, \ldots, x_n \leftarrow \mathbf{let\ rec}\ \mathit{letrec\_bindings}\ \mathbf{in}\ x_n\}\!\}\,(\mathbf{function}\ \mathit{pattern\_matching})}\quad\text{Jrecfun\_letrec}$$

$\boxed{\vdash \mathbf{funval}\,(e)}$ **Function values**

$$\frac{}{\vdash \mathbf{funval}\,((\%\mathbf{prim}\ \mathit{unary\_prim}))}\quad\text{Jfunval\_up}$$

$$\frac{}{\vdash \mathbf{funval}\,((\%\mathbf{prim}\ \mathit{binary\_prim}))}\quad\text{Jfunval\_bp}$$

$$\frac{}{\vdash \mathbf{funval}\,((\%\mathbf{prim}\ \mathit{binary\_prim})\ v)}\quad\text{Jfunval\_bp\_app}$$

$$\frac{}{\vdash \mathbf{funval}\,(\mathbf{function}\ \mathit{pattern\_matching})}\quad\text{Jfunval\_func}$$

$\boxed{\vdash \mathit{unary\_prim}\ \mathit{expr} \xrightarrow{L} \mathit{expr}'}$ **Unary primitive evaluation**

$$\frac{}{\vdash \mathbf{not\ true} \longrightarrow \mathbf{false}}\quad\text{Juprim\_not\_true}$$

$$\frac{}{\vdash \mathbf{not\ false} \longrightarrow \mathbf{true}}\quad\text{Juprim\_not\_false}$$

$$\frac{}{\vdash \sim\!-\ \dot{n} \longrightarrow 0 \mathbin{\dot{-}} \dot{n}}\quad\text{Juprim\_uminus}$$

$$\frac{}{\vdash \mathbf{ref}\ v \xrightarrow{\mathbf{ref}\ v = l} l}\quad\text{Juprim\_ref\_alloc}$$

$$\frac{}{\vdash !\,l \xrightarrow{!\,l = v} v}\quad\text{Juprim\_deref}$$

$\boxed{\vdash \mathit{expr}_1\ \mathit{binary\_prim}\ \mathit{expr}_2 \xrightarrow{L} \mathit{expr}}$ **Binary primitive evaluation**

$$\frac{\vdash \mathbf{funval}\,(v)}{\vdash v = v' \longrightarrow (\%\mathbf{prim\ raise})\,(\mathbf{Invalid\_argument}\,(\mathbf{equal\_error\_string}))}\quad\text{Jbprim\_equal\_fun}$$

$$\frac{}{\vdash \mathit{constant} = \mathit{constant} \longrightarrow \mathbf{true}}\quad\text{Jbprim\_equal\_const\_true}$$

$$\frac{\mathit{constant} \neq \mathit{constant}'}{\vdash \mathit{constant} = \mathit{constant}' \longrightarrow \mathbf{false}}\quad\text{Jbprim\_equal\_const\_false}$$

$$\frac{}{\vdash l = l' \longrightarrow ((\%\mathbf{prim} =)\,((\%\mathbf{prim}\,!)\,l))\,((\%\mathbf{prim}\,!)\,l')}\quad\text{Jbprim\_equal\_loc}$$

$$\frac{}{\vdash (v_1 :: v_2) = (v'_1 :: v'_2) \longrightarrow (((\%\mathbf{prim} =)\,v_1)\,v'_1)\ \&\&\ (((\%\mathbf{prim} =)\,v_2)\,v'_2)}\quad\text{Jbprim\_equal\_cons}$$

$$\frac{}{\vdash (v_1 :: v_2) = [] \longrightarrow \mathbf{false}}\quad\text{Jbprim\_equal\_cons\_nil}$$

$$\frac{}{\vdash [] = (v_1 :: v_2) \longrightarrow \mathbf{false}}\quad\text{Jbprim\_equal\_nil\_cons}$$

$$\frac{\mathbf{length}\,(v_1) \ldots.(v_n) \geq 2}{\vdash (v_1, \ldots., v_n) = (v'_1, \ldots., v'_n) \longrightarrow \mathbf{AND}\,(((\%\mathbf{prim} =)\,v_1)\,v'_1\ \&\& \ldots. \&\&\ ((\%\mathbf{prim} =)\,v_n)\,v'_n)}\quad\text{Jbprim\_equal\_tuple}$$

$$\frac{}{\vdash (\mathit{constr}\,(v_1, \ldots, v_n)) = (\mathit{constr}\,(v'_1, \ldots, v'_n)) \longrightarrow \mathbf{AND}\,(((\%\mathbf{prim} =)\,v_1)\,v'_1\ \&\& \ldots \&\&\ ((\%\mathbf{prim} =)\,v_n)\,v'_n)}\quad\text{Jbprim\_equal\_constr}$$

$$\frac{\mathit{constr} \neq \mathit{constr}'}{\vdash \mathit{constr}\,(v_1, \ldots, v_m) = \mathit{constr}'\,(v'_1, \ldots, v'_n) \longrightarrow \mathbf{false}}\quad\text{Jbprim\_equal\_constr\_false}$$

$$\frac{}{\vdash \mathit{constr}' = \mathit{constr}\,(v_1, \ldots, v_n) \longrightarrow \mathbf{false}}\quad\text{Jbprim\_equal\_const\_constr\_false}$$

$$\frac{}{\vdash \mathit{constr}\,(v_1, \ldots, v_n) = \mathit{constr}' \longrightarrow \mathbf{false}}\quad\text{Jbprim\_equal\_constr\_const\_false}$$

$$\frac{\begin{array}{c}v' = \{\mathit{fn}''_1 = v''_1 ; \ldots ; \mathit{fn}''_m = v''_m\} \\ \mathit{fn}_1 \ldots \mathit{fn}_n\ \mathbf{PERMUTES}\ \mathit{fn}''_1 \ldots \mathit{fn}''_m\end{array}}{\vdash \{\mathit{fn}_1 = v_1 ; \ldots ; \mathit{fn}_n = v_n\} = v' \longrightarrow \mathbf{AND}\,(((\%\mathbf{prim} =)\,v_1)\,(v'.\mathit{fn}_1)\ \&\& \ldots \&\&\ ((\%\mathbf{prim} =)\,v_n)\,(v'.\mathit{fn}_n))}\quad\text{Jbprim\_equal\_rec}$$

$$\frac{}{\vdash \dot{n}_1 + \dot{n}_2 \longrightarrow \dot{n}_1 \mathbin{\dot{+}} \dot{n}_2}\quad\text{Jbprim\_plus}$$

$$\frac{}{\vdash \dot{n}_1 - \dot{n}_2 \longrightarrow \dot{n}_1 \mathbin{\dot{-}} \dot{n}_2}\quad\text{Jbprim\_minus}$$

$$\frac{}{\vdash \dot{n}_1 * \dot{n}_2 \longrightarrow \dot{n}_1 \mathbin{\dot{*}} \dot{n}_2}\quad\text{Jbprim\_times}$$

$$\frac{}{\vdash \dot{n} / 0 \longrightarrow (\%\mathbf{prim\ raise})\ \mathbf{Division\_by\_zero}}\quad\text{Jbprim\_div0}$$

$$\frac{\dot{n}_2 \neq 0}{\vdash \dot{n}_1 / \dot{n}_2 \longrightarrow \dot{n}_1 \mathbin{\dot{/}} \dot{n}_2}\quad\text{Jbprim\_div}$$

$$\frac{}{\vdash l := v \xrightarrow{l := v} ()}\quad\text{Jbprim\_assign}$$

$\boxed{\vdash \mathit{expr}\ \mathbf{with}\ \mathit{pattern\_matching} \longrightarrow \mathit{pattern\_matching}'}$ **Pattern matching step**

$$\frac{\begin{array}{c}\neg(v\ \mathbf{matches}\ \mathit{pat}) \\ \mathbf{length}\,(e_1) \ldots (e_n) \geq 1\end{array}}{\vdash v\ \mathbf{with}\ \mathit{pat} \rightarrow e \mid \mathit{pat}_1 \rightarrow e_1 \mid \ldots \mid \mathit{pat}_n \rightarrow e_n \longrightarrow \mathit{pat}_1 \rightarrow e_1 \mid \ldots \mid \mathit{pat}_n \rightarrow e_n}\quad\text{JRmatching\_next}$$

$\boxed{\vdash \mathit{expr}\ \mathbf{with}\ \mathit{pattern\_matching} \longrightarrow \mathit{expr}'}$ **Pattern matching finished**

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat} \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_m \leftarrow v_m\}\!\}}{\vdash v\ \mathbf{with}\ \mathit{pat} \rightarrow e \mid \mathit{pat}_1 \rightarrow e_1 \mid \ldots \mid \mathit{pat}_n \rightarrow e_n \longrightarrow \{\!\{x_1 \leftarrow v_1, .., x_m \leftarrow v_m\}\!\}\,e}\quad\text{JRmatching\_found}$$

$$\frac{\neg(v\ \mathbf{matches}\ \mathit{pat})}{\vdash v\ \mathbf{with}\ \mathit{pat} \rightarrow e \longrightarrow (\%\mathbf{prim\ raise})\ \mathbf{Match\_failure}}\quad\text{JRmatching\_fail}$$

$\boxed{\vdash \mathit{store} \xrightarrow{L} \mathit{store}'}$ **Store transition**

$$\frac{}{\vdash \mathit{st} \longrightarrow \mathit{st}}\quad\text{JRstore\_empty}$$

$$\frac{\mathit{st}\,(l) \ \rhd\ v}{\vdash \mathit{st} \xrightarrow{!\,l = v} \mathit{st}}\quad\text{JRstore\_lookup}$$

$$\frac{\mathit{st}'\,(l)\ \mathbf{unallocated}}{\vdash \mathit{st}, l \mapsto \mathit{expr}, \mathit{st}' \xrightarrow{l := v} \mathit{st}, l \mapsto \mathbf{remv\_tyvar}\ v, \mathit{st}'}\quad\text{JRstore\_assign}$$

$$\frac{\mathit{st}\,(l)\ \mathbf{unallocated}}{\vdash \mathit{st} \xrightarrow{\mathbf{ref}\ v = l} \mathit{st}, l \mapsto \mathbf{remv\_tyvar}\ v}\quad\text{JRstore\_alloc}$$

$\boxed{\mathit{store}\,(\mathit{location}) \ \rhd\ \mathit{expr}}$ **Store lookup**

$$\frac{\begin{array}{c}\mathit{st}\,(l) \ \rhd\ e' \\ l \neq l'\end{array}}{\mathit{st}, l' \mapsto e\,(l) \ \rhd\ e'}\quad\text{JSstlookup\_rec}$$

$$\frac{}{\mathit{st}, l \mapsto e\,(l) \ \rhd\ e}\quad\text{JSstlookup\_found}$$

$\boxed{\vdash \mathit{expr} \xrightarrow{L} \mathit{expr}'}$ **Expression evaluation**

$$\frac{\vdash \mathit{unary\_prim}\ v \xrightarrow{L} e}{\vdash (\%\mathbf{prim}\ \mathit{unary\_prim})\ v \xrightarrow{L} e}\quad\text{JR\_expr\_uprim}$$

$$\frac{\vdash v_1\ \mathit{binary\_prim}\ v_2 \xrightarrow{L} e}{\vdash ((\%\mathbf{prim}\ \mathit{binary\_prim})\ v_1)\ v_2 \xrightarrow{L} e}\quad\text{JR\_expr\_bprim}$$

$$\frac{}{\vdash (e : t) \longrightarrow e}\quad\text{JR\_expr\_typed\_ctx}$$

$$\frac{\vdash e_0 \xrightarrow{L} e'_0}{\vdash e_1\ e_0 \xrightarrow{L} e_1\ e'_0}\quad\text{JR\_expr\_apply\_ctx\_arg}$$

$$\frac{}{\vdash e\,((\%\mathbf{prim\ raise})\ v) \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_apply\_raise1}$$

$$\frac{\vdash e_1 \xrightarrow{L} e'_1}{\vdash e_1\ v_0 \xrightarrow{L} e'_1\ v_0}\quad\text{JR\_expr\_apply\_ctx\_fun}$$

$$\frac{}{\vdash ((\%\mathbf{prim\ raise})\ v)\ v' \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_apply\_raise2}$$

$$\frac{}{\vdash (\mathbf{function}\ \mathit{pattern\_matching}\ v_0) \longrightarrow \mathbf{match}\ v_0\ \mathbf{with}\ \mathit{pattern\_matching}}\quad\text{JR\_expr\_apply}$$

$$\frac{\vdash e_0 \xrightarrow{L} e'_0}{\vdash \mathbf{let}\ \mathit{pat} = e_0\ \mathbf{in}\ e \xrightarrow{L} \mathbf{let}\ \mathit{pat} = e'_0\ \mathbf{in}\ e}\quad\text{JR\_expr\_let\_ctx}$$

$$\frac{}{\vdash \mathbf{let}\ \mathit{pat} = (\%\mathbf{prim\ raise})\ v\ \mathbf{in}\ e \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_let\_raise}$$

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat} \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_m \leftarrow v_m\}\!\}}{\vdash \mathbf{let}\ \mathit{pat} = v\ \mathbf{in}\ e \longrightarrow \{\!\{x_1 \leftarrow v_1, .., x_m \leftarrow v_m\}\!\}\,e}\quad\text{JR\_expr\_let\_subst}$$

$$\frac{\neg(v\ \mathbf{matches}\ \mathit{pat})}{\vdash \mathbf{let}\ \mathit{pat} = v\ \mathbf{in}\ e \longrightarrow (\%\mathbf{prim\ raise})\ \mathbf{Match\_failure}}\quad\text{JR\_expr\_let\_fail}$$

$$\frac{\begin{array}{l}\mathit{letrec\_bindings} = (x_1 = \mathbf{function}\ \mathit{pattern\_matching}_1\ \mathbf{and} \ldots \mathbf{and}\ x_n = \mathbf{function}\ \mathit{pattern\_matching}_n) \\ \mathbf{recfun}\,(\mathit{letrec\_bindings}, \mathit{pattern\_matching}_1) \ \rhd\ e_1 \quad \ldots \quad \mathbf{recfun}\,(\mathit{letrec\_bindings}, \mathit{pattern\_matching}_n) \ \rhd\ e_n\end{array}}{\vdash \mathbf{let\ rec}\ \mathit{letrec\_bindings}\ \mathbf{in}\ e \longrightarrow \{\!\{x_1 \leftarrow e_1, \ldots, x_n \leftarrow e_n\}\!\}\,e}\quad\text{JR\_expr\_letrec}$$

$$\frac{\vdash e_1 \xrightarrow{L} e'_1}{\vdash e_1 ; e_2 \xrightarrow{L} e'_1 ; e_2}\quad\text{JR\_expr\_sequence\_ctx\_left}$$

$$\frac{}{\vdash ((\%\mathbf{prim\ raise})\ v) ; e \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_sequence\_raise}$$

$$\frac{}{\vdash v ; e_2 \longrightarrow e_2}\quad\text{JR\_expr\_sequence}$$

$$\frac{\vdash e_1 \xrightarrow{L} e'_1}{\vdash \mathbf{if}\ e_1\ \mathbf{then}\ e_2\ \mathbf{else}\ e_3 \xrightarrow{L} \mathbf{if}\ e'_1\ \mathbf{then}\ e_2\ \mathbf{else}\ e_3}\quad\text{JR\_expr\_ifthenelse\_ctx}$$

$$\frac{}{\vdash \mathbf{if}\ (\%\mathbf{prim\ raise})\ v\ \mathbf{then}\ e_1\ \mathbf{else}\ e_2 \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_if\_raise}$$

$$\frac{}{\vdash \mathbf{if\ true\ then}\ e_2\ \mathbf{else}\ e_3 \longrightarrow e_2}\quad\text{JR\_expr\_ifthenelse\_true}$$

$$\frac{}{\vdash \mathbf{if\ false\ then}\ e_2\ \mathbf{else}\ e_3 \longrightarrow e_3}\quad\text{JR\_expr\_ifthenelse\_false}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash \mathbf{match}\ e\ \mathbf{with}\ \mathit{pattern\_matching} \xrightarrow{L} \mathbf{match}\ e'\ \mathbf{with}\ \mathit{pattern\_matching}}\quad\text{JR\_expr\_match\_ctx}$$

$$\frac{}{\vdash \mathbf{match}\ (\%\mathbf{prim\ raise})\ v\ \mathbf{with}\ \mathit{pattern\_matching} \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_match\_raise}$$

$$\frac{\vdash v\ \mathbf{with}\ \mathit{pattern\_matching} \longrightarrow \mathit{pattern\_matching}'}{\vdash \mathbf{match}\ v\ \mathbf{with}\ \mathit{pattern\_matching} \longrightarrow \mathbf{match}\ v\ \mathbf{with}\ \mathit{pattern\_matching}'}\quad\text{JR\_expr\_match\_step}$$

$$\frac{\vdash v\ \mathbf{with}\ \mathit{pattern\_matching} \longrightarrow e'}{\vdash \mathbf{match}\ v\ \mathbf{with}\ \mathit{pattern\_matching} \longrightarrow e'}\quad\text{JR\_expr\_match\_success}$$

$$\frac{}{\vdash e_1\ \&\&\ e_2 \longrightarrow \mathbf{if}\ e_1\ \mathbf{then}\ e_2\ \mathbf{else\ false}}\quad\text{JR\_expr\_and}$$

$$\frac{}{\vdash e_1 \mathbin{||} e_2 \longrightarrow \mathbf{if}\ e_1\ \mathbf{then\ true\ else}\ e_2}\quad\text{JR\_expr\_or}$$

$$\frac{}{\vdash \mathbf{while}\ e_1\ \mathbf{do}\ e_2\ \mathbf{done} \longrightarrow \mathbf{if}\ e_1\ \mathbf{then}\,(e_2 ; \mathbf{while}\ e_1\ \mathbf{do}\ e_2\ \mathbf{done})}\quad\text{JR\_expr\_while}$$

$$\frac{\vdash e_1 \xrightarrow{L} e'_1}{\vdash \mathbf{for}\ x = e_1\ [\mathbf{down}]\mathbf{to}\ e_2\ \mathbf{do}\ e_3\ \mathbf{done} \xrightarrow{L} \mathbf{for}\ x = e'_1\ [\mathbf{down}]\mathbf{to}\ e_2\ \mathbf{do}\ e_3\ \mathbf{done}}\quad\text{JR\_expr\_for\_ctx1}$$

$$\frac{}{\vdash \mathbf{for}\ x = (\%\mathbf{prim\ raise})\ v\ [\mathbf{down}]\mathbf{to}\ e_2\ \mathbf{do}\ e_3\ \mathbf{done} \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_for\_raise1}$$

$$\frac{\vdash e_2 \xrightarrow{L} e'_2}{\vdash \mathbf{for}\ x = v_1\ [\mathbf{down}]\mathbf{to}\ e_2\ \mathbf{do}\ e_3\ \mathbf{done} \xrightarrow{L} \mathbf{for}\ x = v_1\ [\mathbf{down}]\mathbf{to}\ e'_2\ \mathbf{do}\ e_3\ \mathbf{done}}\quad\text{JR\_expr\_for\_ctx2}$$

$$\frac{}{\vdash \mathbf{for}\ x = v\ [\mathbf{down}]\mathbf{to}\ (\%\mathbf{prim\ raise})\ v'\ \mathbf{do}\ e_3\ \mathbf{done} \longrightarrow (\%\mathbf{prim\ raise})\ v'}\quad\text{JR\_expr\_for\_raise2}$$

$$\frac{\dot{n}_1 \mathrel{\dot{\leq}} \dot{n}_2}{\vdash \mathbf{for}\ x = \dot{n}_1\ \mathbf{to}\ \dot{n}_2\ \mathbf{do}\ e\ \mathbf{done} \longrightarrow (\mathbf{let}\ x = \dot{n}_1\ \mathbf{in}\ e) ; \mathbf{for}\ x = \dot{n}_1 \mathbin{\dot{+}} 1\ \mathbf{to}\ \dot{n}_2\ \mathbf{do}\ e\ \mathbf{done}}\quad\text{JR\_expr\_for\_to\_do}$$

$$\frac{\dot{n}_1 \mathrel{\dot{>}} \dot{n}_2}{\vdash \mathbf{for}\ x = \dot{n}_1\ \mathbf{to}\ \dot{n}_2\ \mathbf{do}\ e\ \mathbf{done} \longrightarrow ()}\quad\text{JR\_expr\_for\_to\_done}$$

$$\frac{\dot{n}_2 \mathrel{\dot{\leq}} \dot{n}_1}{\vdash \mathbf{for}\ x = \dot{n}_1\ \mathbf{downto}\ \dot{n}_2\ \mathbf{do}\ e\ \mathbf{done} \longrightarrow (\mathbf{let}\ x = \dot{n}_1\ \mathbf{in}\ e) ; \mathbf{for}\ x = \dot{n}_1 \mathbin{\dot{-}} 1\ \mathbf{downto}\ \dot{n}_2\ \mathbf{do}\ e\ \mathbf{done}}\quad\text{JR\_expr\_for\_downto\_do}$$

$$\frac{\dot{n}_2 \mathrel{\dot{>}} \dot{n}_1}{\vdash \mathbf{for}\ x = \dot{n}_1\ \mathbf{downto}\ \dot{n}_2\ \mathbf{do}\ e\ \mathbf{done} \longrightarrow ()}\quad\text{JR\_expr\_for\_downto\_done}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash \mathbf{try}\ e\ \mathbf{with}\ \mathit{pattern\_matching} \xrightarrow{L} \mathbf{try}\ e'\ \mathbf{with}\ \mathit{pattern\_matching}}\quad\text{JR\_expr\_try\_ctx}$$

$$\frac{}{\vdash \mathbf{try}\ v\ \mathbf{with}\ \mathit{pattern\_matching} \longrightarrow v}\quad\text{JR\_expr\_try\_return}$$

$$\frac{}{\vdash \mathbf{try}\ (\%\mathbf{prim\ raise})\ v\ \mathbf{with}\ \mathit{pat\_exp}_1 \mid \ldots \mid \mathit{pat\_exp}_n \longrightarrow \mathbf{match}\ v\ \mathbf{with}\ \mathit{pat\_exp}_1 \mid \ldots \mid \mathit{pat\_exp}_n \mid \_ \rightarrow ((\%\mathbf{prim\ raise})\ v)}\quad\text{JR\_expr\_try\_catch}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash e_1, .., e_m, e, v_1, .., v_n \xrightarrow{L} e_1, .., e_m, e', v_1, .., v_n}\quad\text{JR\_expr\_tuple\_ctx}$$

$$\frac{}{\vdash e_1, .., e_m, ((\%\mathbf{prim\ raise})\ v), v_1, .., v_n \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_tuple\_raise}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash \mathit{constr}\,(e_1, .., e_m, e, v_1, .., v_n) \xrightarrow{L} \mathit{constr}\,(e_1, .., e_m, e', v_1, .., v_n)}\quad\text{JR\_expr\_constr\_ctx}$$

$$\frac{}{\vdash \mathit{constr}\,(e_1, .., e_m, ((\%\mathbf{prim\ raise})\ v), v_1, .., v_n) \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_constr\_raise}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash e_0 :: e \xrightarrow{L} e_0 :: e'}\quad\text{JR\_expr\_cons\_ctx1}$$

$$\frac{}{\vdash e :: ((\%\mathbf{prim\ raise})\ v) \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_cons\_raise1}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash e :: v \xrightarrow{L} e' :: v}\quad\text{JR\_expr\_cons\_ctx2}$$

$$\frac{}{\vdash ((\%\mathbf{prim\ raise})\ v) :: v' \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_cons\_raise2}$$

$$\frac{\vdash \mathit{expr} \xrightarrow{L} \mathit{expr}'}{\begin{array}{l}\vdash \{\mathit{fn}_1 = e_1 ; \ldots ; \mathit{fn}_m = e_m ; \mathit{field\_name} = \mathit{expr} ; \mathit{fn}'_1 = v_1 ; \ldots ; \mathit{fn}'_n = v_n\} \xrightarrow{L} \\ \quad \{\mathit{fn}_1 = e_1 ; \ldots ; \mathit{fn}_m = e_m ; \mathit{field\_name} = \mathit{expr}' ; \mathit{fn}'_1 = v_1 ; \ldots ; \mathit{fn}'_n = v_n\}\end{array}}\quad\text{JR\_expr\_record\_ctx}$$

$$\frac{}{\vdash \{\mathit{fn}_1 = e_1 ; \ldots ; \mathit{fn}_m = e_m ; \mathit{fn} = (\%\mathbf{prim\ raise})\ v ; \mathit{fn}'_1 = v_1 ; \ldots ; \mathit{fn}'_n = v_n\} \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_record\_raise}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\begin{array}{l}\vdash \{v\ \mathbf{with}\ \mathit{fn}_1 = e_1 ; \ldots ; \mathit{fn}_m = e_m ; \mathit{field\_name} = e ; \mathit{fn}'_1 = v_1 ; \ldots ; \mathit{fn}'_n = v_n\} \xrightarrow{L} \\ \quad \{v\ \mathbf{with}\ \mathit{fn}_1 = e_1 ; \ldots ; \mathit{fn}_m = e_m ; \mathit{field\_name} = e' ; \mathit{fn}'_1 = v_1 ; \ldots ; \mathit{fn}'_n = v_n\}\end{array}}\quad\text{JR\_expr\_record\_with\_ctx1}$$

$$\frac{}{\vdash \{v'\ \mathbf{with}\ \mathit{fn}_1 = e_1 ; \ldots ; \mathit{fn}_m = e_m ; \mathit{fn} = (\%\mathbf{prim\ raise})\ v ; \mathit{fn}'_1 = v_1 ; \ldots ; \mathit{fn}'_n = v_n\} \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_record\_with\_raise1}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash \{e\ \mathbf{with}\ \mathit{field\_name}_1 = e_1 ; \ldots ; \mathit{field\_name}_n = e_n\} \xrightarrow{L} \{e'\ \mathbf{with}\ \mathit{field\_name}_1 = e_1 ; \ldots ; \mathit{field\_name}_n = e_n\}}\quad\text{JR\_expr\_record\_with\_ctx2}$$

$$\frac{}{\vdash \{(\%\mathbf{prim\ raise})\ v\ \mathbf{with}\ \mathit{field\_name}_1 = e_1 ; \ldots ; \mathit{field\_name}_n = e_n\} \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_record\_raise\_ctx2}$$

$$\frac{\begin{array}{c}\mathbf{length}\,(v''_1) \ldots (v''_l) \geq 1 \\ \mathit{field\_name} \notin \mathit{fn}_1 \ldots \mathit{fn}_m\end{array}}{\begin{array}{l}\vdash \{\{\mathit{fn}_1 = v_1 ; \ldots ; \mathit{fn}_m = v_m ; \mathit{field\_name} = v ; \mathit{fn}'_1 = v'_1 ; \ldots ; \mathit{fn}'_n = v'_n\}\ \mathbf{with}\ \mathit{field\_name} = v' ; \mathit{fn}''_1 = v''_1 ; \ldots ; \mathit{fn}''_l = v''_l\} \longrightarrow \\ \quad \{\{\mathit{fn}_1 = v_1 ; \ldots ; \mathit{fn}_m = v_m ; \mathit{field\_name} = v' ; \mathit{fn}'_1 = v'_1 ; \ldots ; \mathit{fn}'_n = v'_n\}\ \mathbf{with}\ \mathit{fn}''_1 = v''_1 ; \ldots ; \mathit{fn}''_l = v''_l\}\end{array}}\quad\text{JR\_expr\_record\_with\_many}$$

$$\frac{\mathit{field\_name} \notin \mathit{fn}_1 \ldots \mathit{fn}_m}{\begin{array}{l}\vdash \{\{\mathit{fn}_1 = v_1 ; \ldots ; \mathit{fn}_m = v_m ; \mathit{field\_name} = v ; \mathit{fn}'_1 = v'_1 ; \ldots ; \mathit{fn}'_n = v'_n\}\ \mathbf{with}\ \mathit{field\_name} = v'\} \longrightarrow \\ \quad \{\mathit{fn}_1 = v_1 ; \ldots ; \mathit{fn}_m = v_m ; \mathit{field\_name} = v' ; \mathit{fn}'_1 = v'_1 ; \ldots ; \mathit{fn}'_n = v'_n\}\end{array}}\quad\text{JR\_expr\_record\_with\_1}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash e\,.\,\mathit{field\_name} \xrightarrow{L} e'\,.\,\mathit{field\_name}}\quad\text{JR\_expr\_record\_access\_ctx}$$

$$\frac{}{\vdash ((\%\mathbf{prim\ raise})\ v)\,.\,\mathit{field\_name} \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_record\_access\_raise}$$

$$\frac{\mathit{field\_name} \notin \mathit{fn}_1 \ldots \mathit{fn}_n}{\vdash \{\mathit{fn}_1 = v_1 ; \ldots ; \mathit{fn}_n = v_n ; \mathit{field\_name} = v ; \mathit{fn}'_1 = v'_1 ; \ldots ; \mathit{fn}'_m = v'_m\}\,.\,\mathit{field\_name} \longrightarrow v}\quad\text{JR\_expr\_record\_access}$$

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash \mathbf{assert}\ e \xrightarrow{L} \mathbf{assert}\ e'}\quad\text{JR\_expr\_assert\_ctx}$$

$$\frac{}{\vdash \mathbf{assert}\,((\%\mathbf{prim\ raise})\ v) \longrightarrow (\%\mathbf{prim\ raise})\ v}\quad\text{JR\_expr\_assert\_raise}$$

$$\frac{}{\vdash \mathbf{assert\ true} \longrightarrow ()}\quad\text{JR\_expr\_assert\_true}$$

$$\frac{}{\vdash \mathbf{assert\ false} \longrightarrow (\%\mathbf{prim\ raise})\ \mathbf{Assert\_failure}}\quad\text{JR\_expr\_assert\_false}$$

$\boxed{\vdash \langle \mathit{definitions}, \mathit{program}\rangle \xrightarrow{L} \langle \mathit{definitions}', \mathit{program}'\rangle}$ **Definition sequence evaluation**

$$\frac{\vdash e \xrightarrow{L} e'}{\vdash \langle \mathit{ds\_value}, \mathbf{let}\ \mathit{pat} = e\,;;\,\mathit{definitions}\rangle \xrightarrow{L} \langle \mathit{ds\_value}, \mathbf{let}\ \mathit{pat} = e'\,;;\,\mathit{definitions}\rangle}\quad\text{Jdefn\_let\_ctx}$$

$$\frac{}{\vdash \langle \mathit{ds\_value}, \mathbf{let}\ \mathit{pat} = (\%\mathbf{prim\ raise})\ v\,;;\,\mathit{definitions}\rangle \longrightarrow \langle \mathit{ds\_value}, (\%\mathbf{prim\ raise})\ v\rangle}\quad\text{Jdefn\_let\_raise}$$

$$\frac{\vdash v\ \mathbf{matches}\ \mathit{pat} \ \rhd\ \{\!\{x_1 \leftarrow v_1, .., x_m \leftarrow v_m\}\!\}}{\vdash \langle \mathit{ds\_value}, \mathbf{let}\ \mathit{pat} = v\,;;\,\mathit{definitions}\rangle \longrightarrow \langle \mathit{ds\_value}, \{\!\{x_1 \leftarrow \mathbf{remv\_tyvar}\ v_1, .., x_m \leftarrow \mathbf{remv\_tyvar}\ v_m\}\!\}\ \mathit{definitions}\rangle}\quad\text{Jdefn\_let\_match}$$

$$\frac{\neg(v\ \mathbf{matches}\ \mathit{pat})}{\vdash \langle \mathit{ds\_value}, \mathbf{let}\ \mathit{pat} = v\,;;\,\mathit{definitions}\rangle \longrightarrow \langle \mathit{ds\_value}, (\%\mathbf{prim\ raise})\ \mathbf{Match\_failure}\rangle}\quad\text{Jdefn\_let\_not\_match}$$

$$\frac{\begin{array}{l}\mathit{letrec\_bindings} = (x_1 = \mathbf{function}\ \mathit{pattern\_matching}_1\ \mathbf{and} \ldots \mathbf{and}\ x_n = \mathbf{function}\ \mathit{pattern\_matching}_n) \\ \mathbf{recfun}\,(\mathit{letrec\_bindings}, \mathit{pattern\_matching}_1) \ \rhd\ e_1 \quad \ldots \quad \mathbf{recfun}\,(\mathit{letrec\_bindings}, \mathit{pattern\_matching}_n) \ \rhd\ e_n\end{array}}{\vdash \langle \mathit{ds\_value}, \mathbf{let\ rec}\ \mathit{letrec\_bindings}\,;;\,\mathit{definitions}\rangle \longrightarrow \langle \mathit{ds\_value}, \{\!\{x_1 \leftarrow \mathbf{remv\_tyvar}\ e_1, \ldots, x_n \leftarrow \mathbf{remv\_tyvar}\ e_n\}\!\}\ \mathit{definitions}\rangle}\quad\text{Jdefn\_letrec}$$

$$\frac{}{\vdash \langle \mathit{ds\_value}, \mathit{type\_definition}\,;;\,\mathit{definitions}\rangle \longrightarrow \langle \mathit{ds\_value}\,;;\,\mathit{type\_definition}, \mathit{definitions}\rangle}\quad\text{Jdefn\_type}$$

$$\frac{}{\vdash \langle \mathit{ds\_value}, \mathit{exception\_definition}\,;;\,\mathit{definitions}\rangle \longrightarrow \langle \mathit{ds\_value}\,;;\,\mathit{exception\_definition}, \mathit{definitions}\rangle}\quad\text{Jdefn\_exn}$$

$\boxed{\vdash \langle \mathit{definitions}, \mathit{program}, \mathit{store}\rangle \longrightarrow \langle \mathit{definitions}', \mathit{program}', \mathit{store}'\rangle}$ **Top-level reduction**

$$\frac{\begin{array}{l}\vdash \mathit{store} \xrightarrow{L} \mathit{store}' \\ \vdash \langle \mathit{definitions\_value}, \mathit{program}\rangle \xrightarrow{L} \langle \mathit{definitions}, \mathit{program}'\rangle\end{array}}{\vdash \langle \mathit{definitions\_value}, \mathit{program}, \mathit{store}\rangle \longrightarrow \langle \mathit{definitions}, \mathit{program}', \mathit{store}'\rangle}\quad\text{JRtop\_defs}$$